Problem 15.30.

Consider an option on a non-dividend-paying stock when the stock price is $30, the exercise price is $29, the risk-free interest rate is 5% per annum, the volatility is 25% per annum, and the time to maturity is four months.

a. *What is the price of the option if it is a European call?*
b. *What is the price of the option if it is an American call?*
c. *What is the price of the option if it is a European put?*
d. *Verify that put–call parity holds.*

In this case $S_0 = 30$, $K = 29$, $r = 0.05$, $\sigma = 0.25$ and $T = 4/12$

$$d_1 = \frac{\ln(30/29) + (0.05 + 0.25^2/2) \times 4/12}{0.25\sqrt{0.3333}} = 0.4225$$

$$d_2 = \frac{\ln(30/29) + (0.05 - 0.25^2/2) \times 4/12}{0.25\sqrt{0.3333}} = 0.2782$$

$$N(0.4225) = 0.6637, \quad N(0.2782) = 0.6096$$

$$N(-0.4225) = 0.3363, \quad N(-0.2782) = 0.3904$$

a. The European call price is

$$30 \times 0.6637 - 29e^{-0.05\times4/12} \times 0.6096 = 2.52$$

or $2.52.

b. The American call price is the same as the European call price. It is $2.52.

c. The European put price is

$$29e^{-0.05\times4/12} \times 0.3904 - 30 \times 0.3363 = 1.05$$

or $1.05.

d. Put-call parity states that:

$$p + S = c + Ke^{-rT}$$

In this case $c = 2.52$, $S_0 = 30$, $K = 29$, $p = 1.05$ and $e^{-rT} = 0.9835$ and it is easy to verify that the relationship is satisfied,

Problem 15.31.

Assume that the stock in Problem 15.30 is due to go ex-dividend in 1.5 months. The expected dividend is 50 cents.

a. *What is the price of the option if it is a European call?*
b. *What is the price of the option if it is a European put?*
c. *If the option is an American call, are there any circumstances when it will be exercised early?*

a. The present value of the dividend must be subtracted from the stock price. This gives a new stock price of:

$$30 - 0.5e^{-0.125\times0.05} = 29.5031$$

and

$$d_1 = \frac{\ln(29.5031/29) + (0.05 + 0.25^2/2) \times 0.3333}{0.25\sqrt{0.3333}} = 0.3068$$

$$d_2 = \frac{\ln(29.5031/29) + (0.05 - 0.25^2/2) \times 0.3333}{0.25\sqrt{0.3333}} = 0.1625$$

$$N(d_1) = 0.6205; \quad N(d_2) = 0.5645$$

The price of the option is therefore

$$29.5031 \times 0.6205 - 29e^{-0.05 \times 4/12} \times 0.5645 = 2.21$$

or \$2.21.

b. Because

$$N(-d_1) = 0.3795, \quad N(-d_2) = 0.4355$$

the value of the option when it is a European put is

$$29e^{-0.05 \times 4/12} \times 0.4355 - 29.5031 \times 0.3795 = 1.22$$

or \$1.22.