Problem 11.2.

What is a lower bound for the price of a four-month call option on a non-dividend-paying stock when the stock price is $28, the strike price is $25, and the risk-free interest rate is 8% per annum?

The lower bound is

$$28-25e^{-0.08\times 0.3333}=\$3.66$$

Problem 11.3.

What is a lower bound for the price of a one-month European put option on a non-dividend-paying stock when the stock price is $12, the strike price is $15, and the risk-free interest rate is 6% per annum?

The lower bound is

$$15e^{-0.06\times 0.08333}-12=\$2.93$$

Problem 11.7.

The price of a non-dividend paying stock is $19 and the price of a three-month European call option on the stock with a strike price of $20 is $1. The risk-free rate is 4% per annum. What is the price of a three-month European put option with a strike price of $20?

In this case, $c=1$, $T=0.25$, $S_0=19$, $K=20$, and $r=0.04$. From put–call parity

$$p=c+Ke^{-rT}-S_0$$

or

$$p=1+20e^{-0.04\times 0.25}-19=1.80$$

so that the European put price is $1.80.

Problem 11.14.

The price of a European call that expires in six months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in two months and again in five months. Interest rates (all maturities) are 10%. What is the price of a European put option that expires in six months and has a strike price of $30?

Using the notation in the chapter, put-call parity [equation (11.10)] gives

$$c+Ke^{-rT}+D=p+S_0$$

or

$$p=c+Ke^{-rT}+D-S_0$$

In this case

$$p=2+30e^{-0.1\times 6/12}+(0.5e^{-0.1\times 2/12}+0.5e^{-0.1\times 5/12})-29=2.51$$

In other words the put price is $2.51.

Problem 11.23.

The prices of European call and put options on a non-dividend-paying stock with 12 months to maturity, a strike price of $120, and an expiration date in 12 months are $20 and $5, respectively. The current stock price is $130. What is the implied risk-free rate?

From put-call parity

$$20+120e^{-r\times 1}=5+130$$

Solving this

$$e^{-r}=115/120$$

so that $r=-\ln(115/120)=0.0426$ or 4.26%

Problem 11.11.
A four-month European call option on a dividend-paying stock is currently selling for $5. The stock price is $64, the strike price is $60, and a dividend of $0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

The present value of the strike price is $60e^{-0.12\times4/12} = \$57.65$. The present value of the dividend is $0.80e^{-0.12\times1/12} = 0.79$. Because

$$5 < 64 - 57.65 - 0.79$$

the condition in equation (11.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64 - 5 = \$59$. The arbitrageur invests $0.79 of this at 12% for one month to pay the dividend of $0.80 in one month. The remaining $58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.
If the stock price declines below $60 in four months, the arbitrageur loses the $5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is $64, has to pay dividends with a present value of $0.79, and closes out the short position when the stock price is $60 or less. Because $57.65 is the present value of $60, the short position generates at least $64 - 57.65 - 0.79 = \$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56 - 5.00 = \$0.56$.
If the stock price is above $60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for $60 in four months and closes out the short position. The present value of the $60 paid for the stock is $57.65 and as before the dividend has a present value of $0.79. The gain from the short position and the exercise of the option is therefore exactly $64 - 57.65 - 0.79 = \$5.56$. The arbitrageur's gain in present value terms is exactly $5.56 - 5.00 = \$0.56$.

Problem 11.24.
A European call option and put option on a stock both have a strike price of $20 and an expiration date in three months. Both sell for $3. The risk-free interest rate is 10% per annum, the current stock price is $19, and a $1 dividend is expected in one month. Identify the arbitrage opportunity open to a trader.

If the call is worth $3, put-call parity shows that the put should be worth

$$3 + 20e^{-0.10\times3/12} + e^{-0.1\times1/12} - 19 = 4.50$$

This is greater than $3. The put is therefore undervalued relative to the call. The correct arbitrage strategy is to buy the put, buy the stock, and short the call. This costs $19. If the stock price in three months is greater than $20, the call is exercised. If it is less than $20, the put is exercised. In either case the arbitrageur sells the stock for $20 and collects the $1 dividend in one month. The present value of the gain to the arbitrageur is

$$-3 - 19 + 3 + 20e^{-0.10\times3/12} + e^{-0.1\times1/12} = 1.50$$