

**Question 41 (24 June 2009, Q1).**

Briefly present Gordon's model for the valuation of common stock.

Hence, consider a stock listed on the Milan Bourse. Suppose it recently paid an annual dividend of €0.25 per share. Assume in the future dividends will grow at a constant yearly rate of 20%. Assume the risk-free rate is 5%, while the expected return on the MIB30 is 15% and the stock's beta is 2. What is the equilibrium price for the stock if the CAPM holds?

**Solution.**

For a presentation of Gordon's model see the textbook.

The expected return on the stock can be derived from the CAPM. It is equal to

$$0.05 + 2 \times 0.10 = 0.25$$

Since the dividends augment by 20% per year, next year we should expect the company to pay a dividend per share equal to

$$0.25 \times 1.20 = 0.30.$$

Considering the hypothesis that dividends will keep growing at the constant rate of 20 percent per year in the future, applying Gordon's formula we find that the stock price should be

$$P = \frac{E[\tilde{D}_1]}{r - g} = \frac{D_0 \times (1 + g)}{r - g} = \frac{0.30}{0.05} = 6.$$

**Question 43 (21 July 2009, Q3).**

Consider the following forecasts on Digicom, an IT firm. It is expected that Digicom will earn €3.00 per share next year (i.e. its *earnings per share* will be €3.00). Today the *book value* of the firm is €15.00 per share. Digicom is to start (from next year) an investment policy which will increase the *book value* of the firm by 10% per year for the foreseeable future. New investment will be entirely financed via retained earnings. It is expected that earnings will increase proportionally to new investment, while the market capitalization rate for Digicom is 20%.

1. Define the following concepts and calculate the corresponding values for Digicom: the *plowback ratio*, the *payout ratio*, the *return on book equity* (ROE).
2. What is the price of Digicom stock if its management will maintain for ever this new investment policy?
3. What would this price be if the management decided to suspend entirely any expansion program and returned all earnings to shareholders? How can you explain the difference?

## Solution.

1. The *plowback ratio* measures the ratio between the retained earnings (per share) and earnings (per share). In the case of an expansion of the production capacity of the firm by 10% per year, it is necessary to invest 10% of the book value of the firm every year. Thus, next year the investment must be equal to 10% of €15 = €1.5 per share.

This implies that, given an estimated value of €3 for next year earnings per share, the *plowback ratio*,  $b$ , is equal to 0.5. The *payout ratio* measures the ratio between dividends per share and earnings per share. In the present context the *payout ratio*,  $p$ , is equal to 0.5. The *return on book equity* measures the return of net capital and is equal to the ratio between earnings per share and the book value (per share). The value for the ROE of Digicom is estimated equal to 20 percent (0.2).

2. To calculate the price of Digicom stock we apply Gordon's formula,

$$P = \frac{E_0[\tilde{D}_1]}{r - g},$$

where  $E_0[\tilde{D}_1]$  is the expected value of dividends paid out by the firm next year, a value equal to  $p \times \text{earnings} = €1.5$ , while  $g$ , the dividends' growth rate, is estimated to be equal to 10 percent. This value is equal to the growth rate for the firm and can be obtained as the product of the *plowback ratio* and the *return on book equity*,  $g = b \times \text{ROE} = 0.5 \times 0.20 = 0.10$ . Hence, the price of Digicom stock is

$$P = \frac{€1.5}{0.20 - 0.10} = €15.$$

3. In the case in which the *plowback ratio* is equal to 0,  $g = b \times \text{ROE} = 0$ , while  $E_0[\tilde{D}_1] = €3$ . Hence, in the case the firm returns to the shareholders all earnings, the price of Digicom stock becomes

$$P = \frac{€3}{0.20} = €15.$$

The stock price is the same. This is because the firm does not present growth opportunities. In fact, the retained earnings when invested yield a return of 20%. This is exactly equal to the market capitalization rate for the firm, which is the opportunity cost of capital for such investment.

**Question 45 (13 October 2010, Q2).**

Telecom Italia is expected to pay a dividend of €2 next year. In line with widespread opinion among financial analysts, this dividend is expected to grow at an annual rate of 2% in the following years.

The interest rate is 3%. Using data from the past 30 years, you estimated the average yearly return on the MIB30 (to be considered the market portfolio) to be 11% and its standard deviation to be 20%.

If over the same period Telecom Italia presented a covariance with the return on the MIB30 of 2%, what is the equilibrium price for Telecom Italia stock?

**Solution.**

To calculate the equilibrium price for Telecom Italia we need applying Gordon's formula, deriving the proper opportunity cost of capital for the firm. Assuming the CAPM holds, we estimate the company's beta,  $\beta$ , equal to

$$\beta = \frac{\text{cov}(\tilde{r}_{\text{MIB30}}, \tilde{r}_{\text{Telecom}})}{\text{var}(\tilde{r}_{\text{MIB30}})} = \frac{0.02}{0.04} = \frac{1}{2}$$

where  $\tilde{r}_{\text{MIB30}}$  and  $\tilde{r}_{\text{Telecom}}$  denote the return on the MIB30 and Telecom Italia stock. From the CAPM, using the estimated values for the stock's beta and for the expected return on the market portfolio,  $E[\tilde{r}_{\text{MIB30}}] = 0.11$ , and also using the interest rate of 3% in lieu of the return on the safe asset, we find that

$$E[\tilde{r}_{\text{Telecom}}] = r_f + \beta (E[\tilde{r}_{\text{MIB30}}] - r_f) = 0.03 + \frac{1}{2} 0.08 = 0.07.$$

Thus, from Gordon's formula we have that

$$P = \frac{E[\tilde{D}_1]}{E[\tilde{r}_{\text{Telecom}}] - g} = \frac{\text{€}2}{0.05} = \text{€}40.$$

**Question 47 (15 January 2013, Q4).**

Americana Airways stock dividend at the end of the year is expected to be \$3 and it is expected to grow at 8% per year for ever. Assume the opportunity cost of capital for Americana Airways stock is 12%

1. What is the economic value for such stock?
2. If the current market price is equal to the economic value, what is next year's expected price?
3. If an investor were to buy Americana Airways stock now and sell it after receiving the \$3 dividend a year from now, what is the expected capital gain (i.e. the price appreciation) in percentage terms? What is the dividend yield, and what would be the holding period return?

**Solution.**

1.

$$P_0 = \frac{D_1}{\kappa - g} = \frac{\$3}{0.12 - 0.08} = \$75.$$

2.

$$P_1 = (1 + g) P_0 = \$75 \times 1.08 = \$81.$$

3. The expected capital gains is  $\$81 - \$75 = \$6.0$ , which in percentage term is equal to  $6/75 = 8\%$ . The dividend yield is equal to  $3/75 = 4\%$ , while to holding period return is  $8\% + 4\% = 12\%$ .