Question 3

Consider the following *zeros*, with nominal values €100 and maturity respectively equal to 1, 2, 3, 4 and 5 years. Suppose their current prices are respectively €97.087, €92.456, €86.384, €79.209 and €71.299.

(a) Calculate the current spot interest rates for the five maturities.

(b) Describe the term structure of interest rates. What is its shape?

(c) Find the one-year forward interest rate valid for next year, and the following three ones.

(d) Assuming the expectation theory holds, which conclusions can we draw on the market expectations of future spot interest rates? Motivate your answer.

HINT:

$$(1 + r_{0,j})^j \, (1 + f_{j,n})^n \equiv (1 + r_{0,j+n})^{j+n}$$

Question 33 (13 September 2010, Q3).

A 5 year coupon bond pays a yearly coupon rate of 8%. Assume that the term structure is flat and the yield rate, y, is 10 percent.

1. What is the bond price, B?
2. What is its modified duration, MD?
3. Estimate the price impact ($\widehat{\Delta B}$) of an increase in the yield rate of 100 basis points, $\Delta y = 1\%$, using the modified duration.
4. Let ΔB denote the actual price impact of an increase in the yield rate of 100 basis point, $\Delta y = 1\%$. What is the approximation error you commit when you estimate the price impact using the modified duration ($\Delta B - \widehat{\Delta B}$)?

HINTS:

$$B = \frac{C}{y}\left[1 - \frac{1}{(1+y)^N}\right] + \frac{P}{(1+y)^N}$$

$$D = \frac{1}{B}\left(\frac{C_1}{1+y} \times 1 + \frac{C_2}{(1+y)^2} \times 2 + \cdots \frac{C_N + P}{(1+y)^N} \times N\right)$$

$$MD = \frac{D}{1+y}.$$

$$\Delta B \approx -MD \times B \times \Delta y$$

$$CO = \frac{1}{B(1+y)^2} \sum_{n=1}^{N} \frac{C_n}{(1+y)^n} (n + n^2)$$

$$\Delta B \approx -MD \times B \times \Delta y + \frac{1}{2} CO \times B \times (\Delta y)^2.$$

Question 31 (24 June 2009, Q3).

Assume the term structure is flat. Then, indicate which and why of the following statements are true and which are false:

1. When, other things being equal, the yield rate rises, so does the duration of a bond.
2. Other things being equal, the larger the coupon rate of a bond and/or smaller its time to maturity, the larger its duration.
3. The modified duration allows to measure exactly the change in the price of a bond induced by either an increase or a reduction in the yield rate.