Problem 5.3.

Suppose that you enter into a six-month forward contract on a non-dividend-paying stock when the stock price is $30 and the risk-free interest rate (with continuous compounding) is 12% per annum. What is the forward price?

The forward price is

$$30e^{0.12\times 0.5} = \$31.86$$

Problem 5.4.

A stock index currently stands at 350. The risk-free interest rate is 8% per annum (with continuous compounding) and the dividend yield on the index is 4% per annum. What should the futures price for a four-month contract be?

The futures price is

$$350e^{(0.08-0.04)\times 0.3333} = \$354.7$$

Problem 5.9.

A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 10% per annum with continuous compounding.

a) *What are the forward price and the initial value of the forward contract?*
b) *Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?*

a) The forward price, F_0, is given by equation (5.1) as:

$$F_0 = 40e^{0.1\times 1} = 44.21$$

or $44.21. The initial value of the forward contract is zero.

b) The delivery price K in the contract is $44.21. The value of the contract, f, after six months is given by equation (5.5) as:

$$f = 45 - 44.21e^{-0.1\times 0.5}$$
$$= 2.95$$

i.e., it is $2.95. The forward price is:

$$45e^{0.1\times 0.5} = 47.31$$

or $47.31.

Problem 5.10.

The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the six-month futures price?

Using equation (5.3) the six month futures price is

$$150e^{(0.07-0.032)\times 0.5} = 152.88$$

or $152.88.

Problem 5.12.

Suppose that the risk-free interest rate is 10% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

The theoretical futures price is

$$400e^{(0.10-0.04)\times 4/12} = 408.08$$

The actual futures price is only 405. This shows that the index futures price is too low relative to the index. The correct arbitrage strategy is

1. Buy futures contracts
2. Short the shares underlying the index.

Problem 5.14.

The two-month interest rates in Switzerland and the United States are, respectively, 1% and 2% per annum with continuous compounding. The spot price of the Swiss franc is $1.0500. The futures price for a contract deliverable in two months is also $1.0500. What arbitrage opportunities does this create?

The theoretical futures price is

(a) $1.0500e^{(0.02-0.01)\times 2/12} = 1.0518$

(b) The actual futures price is too low. This suggests that a Swiss arbitrageur should sell Swiss francs for US dollars and buy Swiss francs back in the futures market.

Problem 5.25.

What is the cost of carry for (a) a non-dividend-paying stock, (b) a stock index, (c) a commodity with storage costs, and (d) a foreign currency?

a) the risk-free rate, b) the excess of the risk-free rate over the dividend yield c) the risk-free rate plus the storage cost, d) the excess of the domestic risk-free rate over the foreign risk-free rate.

Problem 5.28.

The current USD/euro exchange rate is 1.4000 dollar per euro. The six month forward exchange rate is 1.3950. The six month USD interest rate is 1% per annum continuously compounded. Estimate the six month euro interest rate.

If the six-month euro interest rate is r_f then

$$1.3950 = 1.4000e^{(0.01-r_f)\times 0.5}$$

so that

$$0.01 - r_f = 2\ln\left(\frac{1.3950}{1.4000}\right) = -0.00716$$

and $r_f = 0.01716$. The six-month euro interest rate is 1.716%.

Problem 5.29.

The spot price of oil is $80 per barrel and the cost of storing a barrel of oil for one year is $3, payable at the end of the year. The risk-free interest rate is 5% per annum, continuously compounded. What is an upper bound for the one-year futures price of oil?

The present value of the storage costs per barrel is $3e^{-0.05\times 1} = 2.854$. An upper bound to the one-year futures price is $(80+2.854)e^{0.05\times 1} = 87.10$.