Problem 3.7.

A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on a stock index to hedge its risk. The index futures is currently standing at 1080, and each contract is for delivery of $250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?

The formula for the number of contracts that should be shorted gives

$$1.2\times\frac{20{,}000{,}000}{1080\times 250}=88.9$$

Rounding to the nearest whole number, 89 contracts should be shorted. To reduce the beta to 0.6, half of this position, or a short position in 44 contracts, is required.

Problem 3.18.

On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?

A short position in

$$1.3\times\frac{50{,}000\times 30}{50\times 1{,}500}=26$$

contracts is required. It will be profitable if the stock outperforms the market in the sense that its return is greater than that predicted by the capital asset pricing model.

Problem 3.23.

The expected return on the S&P 500 is 12% and the risk-free rate is 5%. What is the expected return on the investment with a beta of (a) 0.2, (b) 0.5, and (c) 1.4?

a) $0.05+0.2\times(0.12-0.05)=0.064$ or 6.4%
b) $0.05+0.5\times(0.12-0.05)=0.085$ or 8.5%
c) $0.05+1.4\times(0.12-0.05)=0.148$ or 14.8%

Problem 3.30.

It is July 16. A company has a portfolio of stocks worth $100 million. The beta of the portfolio is 1.2. The company would like to use the December futures contract on a stock index to change beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 1,000, and each contract is on $250 times the index.

a) *What position should the company take?*
b) *Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?*

a) The company should short

$$\frac{(1.2-0.5)\times 100{,}000{,}000}{1000\times 250}$$

or 280 contracts.

b) The company should take a long position in

$$\frac{(1.5-1.2)\times 100,000,000}{1000\times 250}$$

or 120 contracts.

Problem 3.31. (Excel file)

A fund manager has a portfolio worth $50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next two months and plans to use three-month futures contracts on the S&P 500 to hedge the risk. The current level of the index is 1250, one contract is on 250 times the index, the risk-free rate is 6% per annum, and the dividend yield on the index is 3% per annum. The current 3 month futures price is 1259.

a) *What position should the fund manager take to eliminate all exposure to the market over the next two months?*
b) *Calculate the effect of your strategy on the fund manager's returns if the level of the market in two months is 1,000, 1,100, 1,200, 1,300, and 1,400. Assume that the one-month futures price is 0.25% higher than the index level at this time.*

a) The number of contracts the fund manager should short is

$$0.87\times\frac{50,000,000}{1259\times 250}=138.20$$

Rounding to the nearest whole number, 138 contracts should be shorted.

b) The following table shows that the impact of the strategy. To illustrate the calculations in the table consider the first column. If the index in two months is 1,000, the futures price is 1000×1.0025. The gain on the short futures position is therefore

$$(1259-1002.50)\times 250\times 138=\$8,849,250$$

The return on the index is $3\times 2/12$ =0.5% in the form of dividend and $-250/1250=-20\%$ in the form of capital gains. The total return on the index is therefore -19.5%. The risk-free rate is 1% per two months. The return is therefore -20.5% in excess of the risk-free rate. From the capital asset pricing model we expect the return on the portfolio to be $0.87\times -20.5\%=-17.835\%$ in excess of the risk-free rate. The portfolio return is therefore -16.835%. The loss on the portfolio is $0.16835\times 50,000,000$ or $8,417,500. When this is combined with the gain on the futures the total gain is $431,750.

Index now	1250	1250	1250	1250	1250
Index Level in Two Months	1000	1100	1200	1300	1400
Return on Index in Two Months	-0.20	-0.12	-0.04	0.04	0.12
Return on Index incl divs	-0.195	-0.115	-0.035	0.045	0.125
Excess Return on Index	-0.205	-0.125	-0.045	0.035	0.115
Excess Return on Portfolio	-0.178	-0.109	-0.039	0.030	0.100
Return on Portfolio	-0.168	-0.099	-0.029	0.040	0.110
Portfolio Gain	-8,417,500	-4,937,500	-1,457,500	2,022,500	5,502,500
Futures Now	1259	1259	1259	1259	1259
Futures in Two Months	1002.50	1102.75	1203.00	1303.25	1403.50
Gain on Futures	8,849,250	5,390,625	1,932,000	-1,526,625	-4,985,250
Net Gain on Portfolio	431,750	453,125	474,500	495,875	517,250